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IKKINCHI TARTIBLI CHIZIQLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALARNI TIPINI ANIQLASH

Tursunboy Abdullo o'g'li Qobilov
Chirchiq davlat pedagogika universiteti

Annotatsiya. Xususiy hosilali differensial tenglamalarni qaysi tipga tegishli bo'lishi, uning yuqori tartibli hosilalari oldidagi koeffisiyentlari orqali aniqlanadi. Ushbu maqolada xususiy hosilali differensial tenglamalarning tipini aniqlash bayon qilingan. Kanonik tenglamani yangi noma'lum funksiya kiritish bilan yanada soddaroq ko'rinishga keltirish ko'rsatilgan.

Kalit so'zlar. Differensial tenglama, kvadratik forma va to'la kvadrat.

Аннотация. Тип дифференциальных уравнений с частными производными определяется коэффициентами перед их старшими производными. В данной статье дается определение типа дифференциальных уравнений в частных производных. Показано упрощение канонического уравнения путем введения новой неизвестной функции.

Ключевые слова. Дифференциальное уравнение, квадратичная форма и полный квадрат.

Abstract. The type of differential equations with particular derivatives is determined by the coefficients in front of their higher-order derivatives. This article describes the definition of the type of partial differential equations. A simplification of the canonical equation by introducing a new unknown function is shown.

Key words. Differential equation, quadratic form and full square.

Kirish. $x=(x_1,x_2,...,x_n)\in D\subset E^n$ bo'lib D -ochiq bog'limli soha bo'lsin. R^n -Evkilid fazosi $x_1,x_2,...,x_n$ - ortogonal dekart koordinatalar sistemasidagi x nuqtaning koordinatalari. Tartiblangan manfiy bo'lmagan n ta butun sonning $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ ketma-ketligi n-tartibli mu'lteindeks deyiladi, $|\alpha|=\alpha_1+\alpha_2+...+\alpha_n$ son mu'lteindeksning uyg'unligi deyiladi. $u(x)=u(x_1,x_2,...,x_n)$ funksiyaning $x\in D$ nuqtadagi $|\alpha|=\alpha_1+\alpha_2+...+\alpha_n$ tartibli hosilasini

$$D^\alpha u=D^{\alpha_1}{}_1,D^{\alpha_2}{}_2,\dots,D^{\alpha_n}{}_n u=\frac{\partial^{|\alpha|}u}{\partial^{\alpha_1}x_1,\partial^{\alpha_2}x_2,\dots,\partial^{\alpha_n}x_n},\quad D^\circ u=u(x)$$

Xususiy hoda ko'rinishda belgilashimiz $\alpha=\alpha_i$ bo'lganda

$$D^\alpha u=\frac{\partial^{\alpha_i}u}{\partial x_i^{\alpha_i}},\quad D_i u=\frac{\partial u}{\partial x_i},\quad D_i^2 u=\frac{\partial^2 u}{\partial x_i^2}=u_{x_ix_i}$$

$F=F(x,...p_\alpha,...)$ funksiya D sohada x nuqtaning va $p_\alpha=p_{\alpha_1},p_{\alpha_2},\dots,p_{\alpha_n}=D^\alpha u$,

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$\alpha_i = 0,1,\dots$ haqiqiy o'zgaruvchining berilgan funksiyasi bo'lib, kamida bitta $\dfrac{\partial F}{\partial p_m}$

$m = \max|\alpha|$ hosila noldan farqli bo'lsin.

Asosiy qism. Quyidagi

$$\sum_{i,j=1}^{n} A_{i,j}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} B_i(x)\frac{\partial u}{\partial x_i} + C(x)u = f(x) \qquad (1)$$

ikkinchi tartibli n o'zgaruvchili chiziqli differensial tenglamani R^n Evkilid fazosidagi biror D sohada qaraylik.

Bu yerda $x = (x_1, x_2, \dots, x_n) \in D \subset R^n$, tenglamaning koeffitsientlari $A_{ij}(x), B_i(x), C(x)$ va ozod hadi $f(x)$ yetarlicha silliq berilgan funksiyalar.

Agar $\forall x \in D$ uchun (1) tenglamada $A_{ij}(x) = 0$ bo'lsa , u holda (1) tenglama birinchi tartibli xususiy hosilali differensial tenglama bo'ladi. Shuning uchun qaralayotgan D sohada tenglamaning $A_{ij}(x)$ koeffitsiyentlari bir vaqtda nolga teng bo'lmasin deb talab qilamiz, hamda $A_{ij}(x) = A_{ji}(x)$ tenglik o'rinli bo'lsin.

Faraz qilaylik, $x_0 \in D$ ixtiyoriy nuqta bo'lsin. Chiziqli (1) tenglamaga mos ushbu xaraktristik forma

$$Q(\lambda_1, \dots, \lambda_n) = \sum_{i,j=1}^{n} A_{ij}(x_0)\lambda_i\lambda_j \qquad (2)$$

kvadratik forma deb ataladi.

Algebra kursida ma'lumki, Q kvadratik formani D sohaning har bir x_0 nuqtasida $\lambda_i = \lambda_i(\xi_1, \dots, \xi_n), i = 1, 2, \dots, n$ xosmas almashtirishlar yordamida quyidagi

$$Q = \sum_{i=1}^{n} \alpha_i \xi_i^2 \qquad (3)$$

kanonik ko'rinishiga keltirish mumkin. Bu yerda α_i koeffitsientlar -1,0 va 1 qiymatlarni qabul qiladi.

Qaralayotgan (1) chiziqli tenglamaning klassifikatsiyasi (3) formaning α_i koeffitsientlari qabul qiladigan qiymatlariga asoslanadi.

Agar barcha $\alpha_i = 1$ yoki $\alpha_i = -1, \left(i = \overline{1,n}\right)$ bo'lsa, ya'ni (2) kvadratik forma musbat yoki manfiy aniqlangan bo'lsa, u holda (1) tenglama x_0 nuqtada *elliptik tipdagi tenglama* deyiladi.

Agar α_i koeffitsientlaridan biri manfiy, qolganlari musbat (yoki aksincha) bo'lsa,u holda (1) tenglama x_0 nuqtada *giperbolik tipdagi tenglama* deyiladi.

Agar α_i koeffitsientlaridan kamida bittasi nolga teng bo'lsa, u holda (1) tenglama x_0 nuqtada *parabolik tenglama* deyiladi.

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Agar α_i koeffitsientlarining l $(1 < l < n\text{-}1)$ tasi musbat, qolgan $n\text{-}l$ tasi manfiy bo‘lsa, u holda (1) tenglama x_0 nuqtada *ultragiporbolik tenglama* deyiladi.

Agar D sohaning har bir nuqtasida (3) kvadratik forma koeffitsiyentlarining barchasi noldan farqli va har xil ishorali, barchasi noldan va bir hil ishorali hamda kamida bittasi (hammasi emas) nolga teng bo‘lsa, u holda D sohada mos ravishda *giperbolik,elliptik hamda parabolik tipdagi tenglama* deyiladi.

Agar (1) tenglama qaralayotgan D sohaning turli qismlarida har xil tipga tegishli bo‘lsa, u holda (1) tenglama D sohada *aralash tipdagi tenglama* deyiladi.

1-misol. Butun R^n fazoda aniqlangan n o‘lchovli

$$\Delta u(x) \equiv \sum_{n=1}^{n} \frac{\partial^2 u(x)}{\partial x_i^2} = 0, \qquad (4)$$

Laplas tenglamasiga qaraylik.

Laplas tenglamasiga mos quyidagi

$$Q(\lambda_1,...,\lambda_n) = \sum_{n=1}^{n} \lambda_i^2$$

kvadratik formani tuzamiz.

Agar $i = j$ bo‘lsa, u holda $A_{ij}(x) = 1$ va $i \neq j$ bo‘lganda $A_{ij}(x) = 0$ bo‘ladi. Shuning uchun (3) kvadratik formaning koeffitsiyentlari $\alpha_i = 1,\ (i = \overline{1,n})$ qiymat qabul qiladi.

Demak, Laplas tenglamasi butun R^n fazoda elliptik tipdagi tenglama bo‘ladi.

2-misol. R^{n+1} fazoda aniqlangan n o‘lchovli

$$u(x,t) \equiv u_{tt} - a^2 \sum_{n=1}^{n} \frac{\partial^2 u(x)}{\partial x_i^2} = u_{tt} - a^2 \Delta u = 0, \qquad (5)$$

to‘lqin tarqalish tenglamasini qaraylik. (5) tenglamaga mos kvadratik forma

$Q(\lambda_1,....,\lambda_n,\lambda_{n+1}) = \sum_{n=1}^{n}(-a^2\lambda_i^2) + \lambda_{n+1}^2 = \lambda_{n+1}^2 - \sum_{n=1}^{n} a^2\lambda_i^2$ bo‘ladi.

Bu kvadratik forma $\xi_{n+1}^n = \alpha\lambda_i,\ (i = \overline{1,n}),\ \xi_{n+1} = \lambda_{n+1}$ almashtirish yordamida quyidagi

$$Q = \xi_{n+1}^n - \sum_{n+1}^{n} a^2 \xi_i^2$$

kanonik ko‘rinishida keladi. Bunda α_i koeffitsiyentlardan musbat, qolganlari esa manfiy. Demak, (5) tenglama $R^{n+1}(x,t)$ fazoda giperbolik tipdagi tenglama ekan.

3-misol. $R^{n+1}(x,t)$ fazoda aniqlangan n o‘lchovli

$$u_t - a^2 \sum_{n=1}^{n} \frac{\partial^2 u(x)}{\partial x_i^2} = u_t - a^2 \Delta u = 0, \qquad (6)$$

issiqlik o‘tkazuvchanlik tenglamasini qaraylik.

Bu tenglamaga mos kvadratik forma

$$Q(\lambda_1,....,\lambda_n,\lambda_{n+1}) = -a^2 \sum_{n=1}^{n} \lambda_i^2 + \lambda_{n+1}^2$$

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ko‘rinishida bo‘ladi. Bunda $\alpha_i = -a^2 < 0,\ (i = \overline{1,n})$ va $\alpha_{n+1} = 0$.

Shunday qilib, (6) issiqlik o‘tkazuvchanlik tenglamasi $R^{n+1}_{x,t}$ fazoda parabolik tipdagi tenglama bo‘lar ekan.

Kvadratik formaning musbatligi haqidagi Silvestr alomatiga asosan (2) kvadratik formani (3) kanonik ko‘rinishiga keltirmasdan qaralayotgan xususiy hosilalai differensial tenglamaning tipini aniqlash mumkin.

Xususiy hosilali (1) differensial tenglama elliptik tipda bo‘lishi uchun quyidagi

$$\begin{pmatrix} A_{11}A_{12}\dots\dots\dots A_{1n} \\ A_{21}A_{22}\dots\dots\dots A_{2n} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ A_{n1}A_{n2}\dots\dots\dots A_{nn} \end{pmatrix} \tag{7}$$

Simmetrik matritsaning diagonal minorlari musbat aniqlangan bo‘lishi zarur va yetarli.

Agar (1) tenglama ikki o‘zgaruvchili $x_1 = x,\ x_2 = y$ bo‘lsa, uni quyidagi

$$a(x,y)u_{xx} + 2b(x,y)u_{xy} + c(x,y)u_{yy} = F(x,y,u,u_x,u_y), \tag{8}$$

ko‘rinishda ifodalash mumkin.

Bu tenglamaga mos kvadratik forma

$$Q(\lambda_1,\lambda_2) = a(x,y)\lambda_1^2 + 2b(x,y)\lambda_1\lambda_2 + c(x,y)\lambda_2^2 \tag{9}$$

bo‘ladi.

Agar $a(x,y) \neq 0$ bo‘lsa, u holda (9) kva,dratik formani ushbu

$$Q(\lambda_1,\lambda_2) = a\left(\lambda_1 + \frac{b}{a}\lambda_2\right)^2 - \frac{b^2 - ac}{a}\lambda_2^2,\ Q(\lambda_1,\lambda_2) = a\left(\lambda_1 + \frac{b}{a}\lambda_2\right)^2 - \frac{b^2 - ac}{a}\lambda_2^2, \tag{10}$$

ko‘rinishda ifodalash mumkin.

Agar $\delta(M_0) = b^2 - ac < 0$ bo‘lsa, u holda (9) kvadratik forma $M_0 = (x_0, y_0)$ nuqtada musbat yoki manfiy aniqlangan bo‘ladi.

Chunki (10) ifoda quyidagi almashti'rish

$$\xi_1 = \sqrt{|a|}\left(\lambda_1 + \frac{b}{a}\lambda_2\right), \qquad \xi_2 = \sqrt{\frac{ac - b^2}{|a|}}\lambda_2$$

yordamida ushbu kanonik ko‘rinishga

$$Q = \begin{cases} \xi_1^2 + \xi_2^2, a > 0, \\ -\xi_1^2 - \xi_2^2, a < 0 \end{cases}$$

keladi. Bundan $\delta(M_0) = b^2 - ac < 0$ bo‘lganda (8) tenglamaning $M_0 = (x_0, y_0)$ nuqtada elliptik tipda bodishi kelib chiqadi.

Faraz qilaylik, $M_0 = (x_0, y_0)$ nuqtada $\delta(M_0) = b^2 - ac < 0$ bo‘lsin. U holda (10) kvadratik forma

$$\xi_1 = \sqrt{|a|}\left(\lambda_1 + \frac{b}{a}\lambda_2\right), \qquad \xi_2 = \sqrt{\frac{ac-b^2}{|a|}}\lambda_2$$

almashtirishdan keyin

$$Q = \begin{cases} \xi_1^2 + \xi_2^2, a > 0, \\ -\xi_1^2 - \xi_2^2, a < 0 \end{cases}$$

ko‘rinishga keladi, ya’ni (3) kvadratik formaning koeffitsiyentlari α_1 va α_2 har xil ishorali. Bundan ko‘rinadiki, qaralayotgan (8) tenglama $M_0 = (x_0, y_0)$ nuqtada giperbolik tipga tegishli ekan.

Agar $M_0 = (x_0, y_0)$ nuqtada $\delta(M_0) = b^2 - ac = 0$ bo‘lsa, u holda

$$\xi_1 = \sqrt{|a|}\left(\lambda_1 + \frac{b}{a}\lambda_2\right), \qquad \xi_2 = \lambda_2$$

xosmas almashtirish (10) kvadratik formani quyidagi

$$Q = \begin{cases} \xi_1^2 + \xi_2^2, a > 0, \\ -\xi_1^2 - \xi_2^2, a < 0 \end{cases}$$

ko‘rinishga keltiradi. Demak, (3) kvadratik formaning α_1 va α_2 koeffitsiyentlaridan biri nolga teng, ikkinchisi noldan farqli, ya’ni qaralayotgan (8) tenglama $M_0 = (x_0, y_0)$ nuqtada parabolik tipdagi tenglama ekan.

Yuqoridagi mulohazalardan qaralayotgan xususiy hosilali (8) differensial tenglamaning tipini aniqlash $\delta(M_0) = b^2 - ac$ diskriminantning ishorasiga bog‘liq ekanligi ko‘rinadi.

Agar biror $M_0 = (x_0, y_0)$ nuqtada $\delta(M_0) = b^2 - ac > 0$ bo‘lsa, u holda (8) tenglama M_0 *nuqtada giperbolik tipdagi tenglama* deyiladi.

Agar biror $M_0 = (x_0, y_0)$ nuqtada $\delta(M_0) = b^2 - ac < 0$ bo'lsa, u holda (8) tenglama M_0 *nuqtada elliptik tipdagi tenglama* deyiladi.

Agar biror $M_0 = (x_0, y_0)$ nuqtada $\delta(M_0) = b^2 - ac = 0$ bo‘lsa, u holda (8) tenglama shu nuqtada *parabolik tipdagi tenglama* deyiladi.

4 – misol. Quyidagi tenglamalarning tipini aniqlang.

a) $u_{xx} - 4u_{xy} + 2u_{xz} + 4u_{yy} + u_{zz} + 3xyu = 0;$

b) $u_{xx} + u_{xy} - 25u_{yz} + 2u_{yy} + 6u_{zz} + xu_z + x^2yu = 0;$

c) $4u_{xx} + 6u_{xy} + 2u_{yy} + 10u_{xz} + 4u_{yz} - 6u_{zz} = 0;$

Yechish. Berilgan tenglamalarning yuqori tartibli hosilalari oldidagi koeffitsiyentlari o‘zgarmas. Shuning uchun bu tenglamalarning tipi butun fazoda aniqlanadi.

a) Berilgan tenglamaga mos xarakteristik forma

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$$Q(\lambda_1,\lambda_2,\lambda_3)=\lambda_1^2-4\lambda_1\lambda_2+2\lambda_1\lambda_3+4\lambda_2^2+\lambda_3^2=$$
$$=(\lambda_1-2\lambda_2+\lambda_3)^2+(\lambda_2+\lambda_3)^2-(\lambda_2-\lambda_3)^2$$

ko‘rinishda bo‘ladi. Quyidagi

$$\lambda_1=\xi_1+\frac{1}{2}\xi_2+\frac{3}{2}\xi_3;\quad \lambda_1=\frac{1}{2}(\xi_2+\xi_3);\quad \lambda_3=\frac{1}{2}(\xi_2-\xi_3)$$

xosmas almashtirish yordamida $Q(\lambda_1, \lambda_2, \lambda_3)$ forma

$$K(\xi_1, \xi_2, \xi_3)=\xi_1^2+\xi_2^2-\xi_3^2$$

kanonik ko’rinishga keladi.

$\alpha_1=\alpha_2=1, \alpha_3=-1$ koeffitsiyentlar noldan farqli va har xil ishorali. Ta'rifga asosan bu tenglama giperbolik tipga tegishli.

b) Yuqoridagi kabi berilgan differensial tenglamaning xarakteristik fo rmasini tuzamiz

$$Q(\lambda_1, \lambda_2, \lambda_3)=\lambda_1^2+2\lambda_1\lambda_2-2\lambda_1\lambda_3+2\lambda_2^2+6\lambda_3^2 \qquad (*)$$

va uni to‘la kvadratlarga ajratib, kanonik ko'rinishga keltiramiz:

$$Q=\lambda_1^2+2\lambda_1\lambda_2-2\lambda_1\lambda_3-2\lambda_2\lambda_3+\lambda_2^2+\lambda_3^2+\lambda_2^2+2\lambda_2\lambda_3+\lambda_3^2+4\lambda_3^2=(\lambda_1+\lambda_2-\lambda_3)^2+(\lambda_2+\lambda_3)^2+4\lambda_3^2.$$

Bundan quyidagi xosmas almashtirishlar natijasida

$$\xi_1=\lambda_1+\lambda_2-\lambda_3,\qquad \xi_2=\lambda_2+\lambda_3,\qquad \xi_3=2\lambda_3.$$

Q kvadratik forma ushbu

$$K(\xi_1, \xi_2, \xi_3)=\xi_1^2+\xi_2^2+\xi_3^2$$

kanonik ko'rinishga keladi.

Demak, (3) kvadratik formaning $\alpha_1,$ α_2 va α_3 koeffitsiyentlari noldan farqli va bir xil ishorali. Shuning uchun berilgan tenglama R^3 fazoda elliptik tipda bo’ladi.

c)tenglamaga mos xarakteristik forma

$$Q(\lambda_1,\lambda_2,\lambda_3)=4\lambda_1^2+6\lambda_1\lambda_2+2\lambda_2^2+10\lambda_1\lambda_3+4\lambda_2\lambda_3-6\lambda_3^2=\frac{1}{4}(4\lambda_1+3\lambda_2+5\lambda_3)^2-\frac{1}{4}(\lambda_2+7\lambda_3)^2$$

quyidagi

$$\lambda_1=\frac{1}{2}\zeta_1-\frac{3}{2}\zeta_2+4\zeta_3;\quad \lambda_2=2\zeta_2-7\zeta_3;\quad \lambda_3=\zeta_3 \qquad \Delta_i, i=1,2,3,$$

xosamas almashtirish yordamida

$$K(\zeta_1,\zeta_2,\zeta_3)=\zeta_1^2-\zeta_2^2+0\zeta_3^2$$

kanonik ko'rinishga keladi.

Bundan ko'rinadiki, $\alpha_1=1, \alpha_2=-1, \alpha_3=0$ ekan. Demak, berilgan tenglama parobolik tipga tegishli ekan.

FOYDALANILGAN ADABIYOTLAR

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